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hintreg: An R package for fitting a heterogeneous normal interval regression model to interval data with an indifference limen[1]

Yasutomo Murasawa

Abstract

Interval data with an indifference limen arise when a survey asks about a *change* of a continuous variable and the respondents choose from a set of given intervals including 'no change'. This vignette introduces **hintreg**, an R package for fitting a heterogeneous normal interval regression model to such data. The model allows for heterogeneity not only in the location and scale parameters, but also in the lower and upper thresholds of the indifference limen, thus reducing the risk of model misspecification. An analysis of survey data on inflation expectations illustrates how to use the package.

Keywords: difference limen, heterogeneity, inflation expectation, ML estimation, survey data

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(1) This package was developed during my visit to the Department of International Business, National Chengchi University, in the summer of 2022. I thank Natsuki Arai and Biing-Shen Kuo for their hospitality and support, and Konan University for fund-

Introduction

Survey questions about a continuous variable (e.g., income) often ask respondents to choose from a set of given intervals. Moreover, for questions about a *change* of a continuous variable (e.g., inflation, which is a change in the price level), the choice may include a category 'no change'. This category does not literally mean a 0% change, whose probability is 0 for a continuous random variable, but implies an *indifference limen*, which allows for an unnoticeable change. With known thresholds except for an indifference limen, interval data with an indifference limen differ from both interval and ordinal data. Standard econometric software packages such as gretl and Stata do not have commands to analyze such data easily.

Conceptually, fitting a regression model to interval data with an indifference limen is straightforward. Just write down the likelihood function and maximize it with respect to the parameters. See Murasawa (2013) for an example.[2] One can also use an R package **oglmx** by Carroll (2018) to fit an ordered response model with some known thresholds (=interval regression model with some unknown thresholds), even allowing for conditional heteroskedasticity. However, **oglmx** does not allow for heterogeneous thresholds.[3]

This vignette introduces **hintreg**, an R package for fitting a heterogeneous nor-

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ing the visit. This work was also supported by JSPS KAKENHI Grant Number JP21K01434.

(2) Murasawa (2020) applies a Bayesian method, assuming a normal mixture model.

(3) Some R packages for fitting an ordered response model allow for heterogeneous thresholds; e.g., **hopit** by Dańko (2019) and **ordinal** by Christensen (2019). They typically do not allow for known thresholds, however; i.e., they cannot fit an interval regression model.

mal interval regression model to interval data with an indifference limen. The model allows for heterogeneity not only in the location and scale parameters, but also in the lower and upper thresholds of the indifference limen. Allowing for heterogeneity in the indifference limen is not only interesting by itself, but also reduces the risk of model misspecification, which is important because for a nonlinear model such as an interval regression model, ignoring any kind of heterogeneity leads to inconsistency of the ML estimator.

The vignette proceeds as follows. Section 2 defines a heterogeneous interval regression model for interval data with an indifference limen. Section 3 explains how to install **hintreg** and use its main function `hintreg()`. Section 4 analyzes survey data on inflation expectations to illustrate how to use **hintreg**.

I Heterogeneous interval regression model

1 Normal interval regression model

Let $(\mathbf{y}, \mathbf{X})$ be a $(1+k)$-variate random sample of size n, where y_i is an ordered factor with $J+1$ levels. A *normal interval regression model* for y_i given $\mathbf{x}_i$ is for $i=1, \ldots, n$,

$$y_i := \begin{cases} 0 & \text{if } c_0 < y_i^* \leq c_1 \\ \vdots & \\ J & \text{if } c_J < y_i^* \leq c_{J+1} \end{cases}$$

$$y_i^* | \mathbf{x}_i \sim \mathrm{N}(\mathbf{x}_i'\boldsymbol{\beta}, \sigma^2)$$

where we know $\{c_j\}$. This model is standard. One can use `intreg` command in gretl and Stata to fit this model.

2 Conditional heteroskedasticity

A *generalized normal interval regression model* assumes a generalized normal linear regression model for y_i^* given $\mathbf{x}_i$, i.e., for $i=1, \ldots, n$,

$$y_i^* | \mathbf{x}_i \sim \mathrm{N}(\mathbf{x}_i'\boldsymbol{\beta}, \exp(2\mathbf{x}_i'\boldsymbol{\delta}))$$

(4)

This model is also standard. One can use `intreg` command in Stata to fit this model.

3 Indifference limen

Assume that for $i=1, \ldots, n$,

$$y_i := \begin{cases} 0 & \text{if } \gamma_0 < y_i^* \leq \gamma_1 \\ \vdots & \\ J & \text{if } \gamma_J < y_i^* \leq \gamma_{J+1} \end{cases}$$

$$y_i^* | \mathbf{x}_i \sim \mathrm{N}(\mathbf{x}_i'\boldsymbol{\beta}, \exp(2\mathbf{x}_i'\boldsymbol{\delta}))$$

where $\gamma_0 < \cdots < \gamma_l < 0 < \gamma_u < \cdots < \gamma_{J+1}$ and we know $\{\gamma_j\}$ except for $[\gamma_l, \gamma_u]$, which is an indifference limen. This model is not standard. However, one can use an R package **oglmx** to fit this model.

4 Heterogeneous indifference limen

The indifference limen $[\gamma_l, \gamma_u]$ may differ across i. Let for $i=1, \ldots, n$,

$$\gamma_{l,i} := \bar{\gamma}_l \Lambda(\mathbf{x}_i'\boldsymbol{\lambda})$$

$$\gamma_{u,i} := \bar{\gamma}_u \Lambda(\mathbf{x}_i'\boldsymbol{\upsilon})$$

where $\bar{\gamma}_l \in [\gamma_{l-1}, 0)$ is a lower bound on $\{\gamma_{l,i}\}$, $\bar{\gamma}_u \in (0, \gamma_{u+1}]$ is an upper bound on $\{\gamma_{u,i}\}$, and $\Lambda(.)$ is the logistic function. This model is not standard. We name it a *heterogeneous normal interval regression model*. One can use our R package **hintreg** to fit this model.

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(4) Following `intreg` command in Stata, we model conditional heteroskedasticity not in variance but in standard deviation.

II hintreg package

1 Installation

hintreg is currently available from the author's repository on Github. To install a package from Github, it is convenient to use `install_github()` in **devtools**. So one can start using **hintreg** in the following four steps:

1. install **devtools** from CRAN
2. load and attach **devtools**
3. install **hintreg** from Github
4. load and attach **hintreg**

or simply by running the following code:

```
install.packages("devtools")
library(devtools)
install_github("ystmmrsw/hintreg")
library(hintreg)
```

2 Available functions

hintreg has only three functions: `intreg()`, `gintreg()`, and `hintreg()`. The first two functions are not new. Given known thresholds, they fit normal and generalized normal interval regression models, respectively. Here we introduce our main function `hintreg()`.

`hintreg()` takes the following arguments:

```
hintreg(
  location,
  scale,
  lower,
  upper,
  data,
  start,
```

```
  weights,
  threshbelow,
  threshabove,
  limenlb,
  limenub
)
```

Among these arguments, `data`, `start`, and `weights` are standard. So we do not discuss them here.

The first four arguments are all formulas:

`location` specifies the mean; e.g., `y ~ x`

`scale` specifies the standard deviation; e.g., `~ x`

`lower` specifies the lower threshold of the indifference limen; e.g., `~ x`

`upper` specifies the upper threshold of the indifference limen; e.g., `~ x`

The left-hand side of the formula in `location` is the dependent variable, which must be an ordered factor. The left-hand sides of the other formulas are empty.

The last four arguments are parameters for the thresholds and the indifference limen:

`threshbelow` is a vector of known thresholds below the indifference limen

`threshabove` is a vector of known thresholds above the indifference limen

`limenlb` is a lower bound on the heterogeneous indifference limen

`limenub` is an upper bound on the heterogeneous indifference limen

If the dependent variable has $J+1$ levels with J thresholds, then there must be $J-2$ elements in `threshbelow` and `threshabove` (one of them can be null), excluding the thresholds of the indifference limen. If not null, then the elements in `threshbelow` and `threshabove` must be negative and positive, respectively, both in the increasing order. Identification requires that $J \geq 4$. To maintain the order of thresholds, `limenlb` must be no less than `max(threshbelow)` and negative, and `limenub` must be positive and no greater than `min(threshabove)`.

Internally, `hintreg()` maximizes the log-likelihood function using `optim()`, where we use the BFGS method and numerical derivatives for the gradients.

III　An illustration

1　Survey data on inflation expectations

hintreg has a data set `ias2009febmay` taken from the Bank of England Inflation Attitudes Survey.[5] This data set allows us to replicate a result in Blanchflower and MacCoille (2009), who study the formation of public inflation expectations in the UK.

The Inflation Attitudes Survey is a quarterly survey starting from February 2001, and collects a quota sample of adults throughout the UK. The quota is 4,000 in February, and 2,000 in May, August, and November. The survey asks both inflation perceptions and expectations, and the respondents choose from the following eight intervals (excluding 'No idea'):

1. Gone/Go down
2. Not changed/change
3. Up by 1% or less
4. Up by 1% but less than 2%
5. Up by 2% but less than 3%
6. Up by 3% but less than 4%
7. Up by 4% but less than 5%

(5) The data set of the Inflation Attitudes Survey is available at `https://www.bankofengland.co.uk/statistics/research-datasets`.

8. Up by 5% or more

So we have interval data with an indifference limen (choice 2).

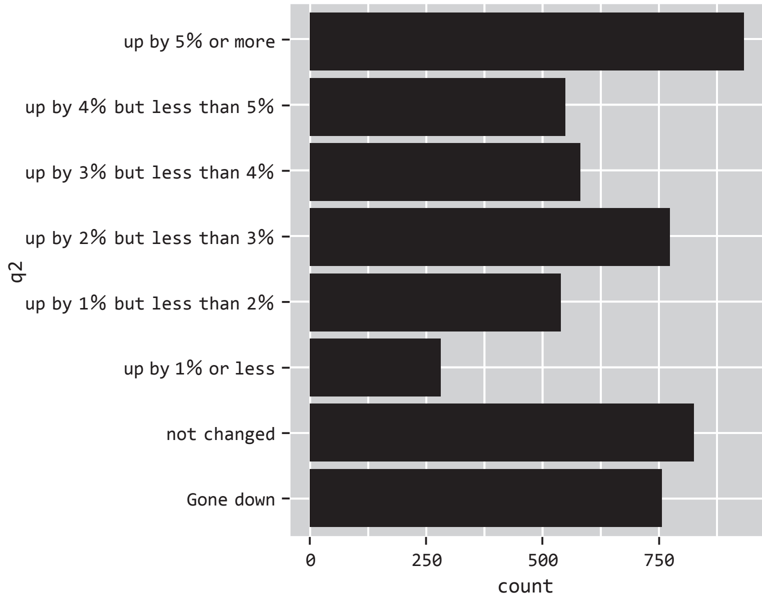
The survey dates and variables in `ias2009febmay` are just enough to replicate Table 3, column (3) in Blanchflower and MacCoille (2009). One can check the variables in the data set by summarizing it:

```
summary(ias2009febmay)
#>                            q2           sex           age
#>   up by 5% or more          :932  female:2680  15-24: 597
#>   not changed               :824  male  :2563  25-34: 913
#>   up by 2% but less than 3%:776               35-44:1043
#>   Gone down                 :757               45-54: 980
#>   up by 3% but less than 4%:581               55-64: 753
#>   up by 4% but less than 5%:552               65+  : 957
#>   (Other)                   :821
#>                  work                      educ
#>   not working       :2340  low (GCSE)           :1174
#>   full or part time:2903      medium (A-Level):2793
#>                               high (Degree)    :1276
#>
#>
#>
#>
#>                                     tenure
#>   owned outright                        :1433
#>   mortgage                              :1970
#>   council rent                          : 667
#>   other (inc private rent pre-2016):1173
#>
#>
#>
```

The variable `q2` is the respondent's answer to the question about inflation expectation over the next 12 months. One can visualize its distribution among the respondents using a bar chart:

```
ggplot(ias2009febmay) +
```

```
  geom_bar(aes(x = q2)) +
  coord_flip()
```



2 Normal interval regression model

Blanchflower and MacCoille (2009) implicitly assume that the indifference limen is [−.5, .5], and fit normal interval regression models to interval data on inflation expectations. Since `hintreg()` assumes unknown thresholds for the indifference limen, we cannot use `hintreg()` here. Using `intreg()` instead, one can replicate Table 3, column (3) in Blanchflower and MacCoille (2009) as follows:

```
intreg(
  q2 ~ age + work + sex + educ + tenure,
  data       = ias2009febmay,
  thresholds = c(-.5, .5, 1, 2, 3, 4, 5)
```

```
) |> summary()
#> Call:
#> intreg (formula = q2 ~ age + work + sex + educ + tenure, data = ias2009
febmay,
#>     thresholds = c (-0.5, 0.5, 1, 2, 3, 4, 5))
#>
#> No. of iterations: 75
#> log-likelihood: -10679.33
#> AIC: 21386.67
#>
#> Parameters:
#>                                              Estimate Std. Error  t value
#> (Intercept)                                   2.61294   0.19692   13.269
#> age25-34                                     -0.06533   0.15563   -0.420
#> age35-44                                     -0.07041   0.15371   -0.458
#> age45-54                                      0.17302   0.15574    1.111
#> age55-64                                      0.28603   0.17055    1.677
#> age65+                                        0.25534   0.17914    1.425
#> workfull or part time                         0.02986   0.10049    0.297
#> sexmale                                      -0.19222   0.08122   -2.367
#> educmedium (A-Level)                         -0.22867   0.11090   -2.062
#> educhigh (Degree)                            -0.73437   0.13002   -5.648
#> tenuremortgage                               -0.36224   0.12117   -2.989
#> tenurecouncil rent                            0.62048   0.14806    4.191
#> tenureother (inc private rent pre-2016)       0.11986   0.13320    0.900
#> log s.d.                                      1.02872   0.01304   78.913
#>                                             Pr(>|t|)
#> (Intercept)                                  < 2e-16 ***
#> age25-34                                      0.6746
#> age35-44                                      0.6469
#> age45-54                                      0.2666
#> age55-64                                      0.0935 .
#> age65+                                        0.1541
#> workfull or part time                         0.7663
#> sexmale                                       0.0179 *
#> educmedium (A-Level)                          0.0392 *
#> educhigh (Degree)                           1.62e-08 ***
#> tenuremortgage                                0.0028 **
```

```
#> tenurecouncil rent                    2.78e-05 ***
#> tenureother (inc private rent pre-2016)    0.3682
#> log s.d.                               < 2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We want to introduce conditional heteroskedasticity and a heterogeneous indifference limen into this model. However, ML estimation of such a general model with many explanatory variables takes too much time for an illustration. So we include only `sex` as an explanatory variable below; i.e., we measure a gender gap in inflation expectations without controlling for other variables.(6)

```
lout <- intreg(
  q2 ~ sex,
  data       = ias2009febmay,
  thresholds = c (-.5, .5, 1, 2, 3, 4, 5)
)
summary(lout)
#> Call:
#> intreg (formula = q2 ~ sex, data = ias2009febmay, thresholds = c(-0.5,
#>    0.5, 1, 2, 3, 4, 5))
#>
#> No. of iterations: 51
#> log-likelihood: -10755.11
#> AIC: 21516.23
#>
#> Parameters:
#>              Estimate  Std. Error t value  Pr(>|t|)
#> (Intercept)   2.40023    0.05702  42.098   <2e-16 ***
#> sexmale      -0.21066    0.08156  -2.583   0.0098 **
#> log s.d.      1.04500    0.01305  80.104   <2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We see that for the UK in February and May 2009, men on average had signifi-

(6) Of course, interested readers can try including more explanatory variables by themselves.

cantly lower inflation expectations than women by about .2%.

3 Generalized normal interval regression model

Now we introduce conditional heteroskedasticity. Since the likelihood function may no longer be unimodal, one should choose the starting values for the parameters carefully, though the default starting values may work. Here we set the starting values as follows:

mean equation: use the estimated coefficients in a normal interval regression model

standard deviation equation: use the estimated log standard deviation of the error term in a normal interval regression model as the intercept; set the other coefficients to 0

One can implement this procedure as follows:

```
vstart_beta  <- coef(lout)
ck           <- length(vstart_beta)
vstart_delta <- c(lout$logscale, numeric(ck - 1))
vstart       <- c(vstart_beta, vstart_delta)
```

Since `hintreg()` assumes unknown thresholds for the indifference limen, we cannot use `hintreg()` here. Using `gintreg()` instead, we can fit a generalized normal interval regression model to interval data as follows:

```
lout <- gintreg(
  q2 ~ sex,
     ~ sex,
  data       = ias2009febmay,
  start      = vstart,
  thresholds = c(-.5, .5, 1, 2, 3, 4, 5)
)
summary(lout)
#> Call:
```

```
#> gintreg(location = q2 ~ sex, scale = ~sex, data = ias2009febmay,
#>     start = vstart, thresholds = c (-0.5, 0.5, 1, 2, 3, 4, 5))
#>
#> No. of iterations: 43
#> log-likelihood: -10755.00
#> AIC: 21518.01
#>
#> Location coefficients:
#>              Estimate  Std. Error t value  Pr(>|t|)
#> (Intercept)  2.40003    0.05671  42.323  < 2e-16 ***
#> sexmale     -0.21035    0.08161  -2.577  0.00995 **
#>
#> Scale coefficients:
#>              Estimate  Std. Error t value  Pr(>|t|)
#> (Intercept)  1.03929    0.01814   57.28   <2e-16 ***
#> sexmale      0.01175    0.02611    0.45    0.653
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We see that for the UK in February and May 2009, the standard deviations of inflation expectations among men and women were not significantly different.

4 Estimating the indifference limen

Blanchflower and MacCoille (2009) implicitly assume that the indifference limen is $[\gamma_l, \gamma_u] := [-.5, .5]$. We can use `hintreg()` to estimate $[\gamma_l, \gamma_u]$ by restricting the threshold equations to have only intercepts. Here we set the starting values as follows:

mean equation: use the estimated coefficients for the mean equation in a generalized normal interval regression model

standard deviation equation: use the estimated coefficients for the standard deviation equation in a generalized normal interval regression model

thresholds: set the intercepts to 0, meaning that the starting values for γ_l and γ_u are $\bar{\gamma}_l/2$ and $\bar{\gamma}_h/2$, respectively

One can implement this procedure as follows:

```
vstart_beta  <- lout$coefL
vstart_delta <- lout$coefS
vstart_gamma <- numeric(2)
vstart       <- c(vstart_beta, vstart_delta, vstart_gamma)
```

The vectors of known thresholds below and above the indifference limen are null and (1, ..., 5), respectively. It is natural to set $\bar{\gamma}_h := 1$. By symmetry, we set $\bar{\gamma}_l := -1$, though it can be any negative number. We can fit a heterogeneous normal interval regression model to interval data with an indifference limen as follows:

```
lout <- hintreg(
  q2 ~ sex,
    ~ sex,
    ~ 1,
    ~ 1,
  data        = ias2009febmay,
  start       = vstart,
  threshbelow = NULL,
  threshabove = 1:5,
  limenlb     = -1,
  limenub     = 1
)
summary(lout)
#> Call:
#> hintreg(location = q2 ~ sex, scale = ~sex, lower = ~1, upper = ~1,
#>     data = ias2009febmay, start = vstart, threshbelow = NULL,
#>     threshabove = 1:5, limenlb = -1, limenub = 1)
#>
#> No. of iterations: 74
#> log-likelihood: -10671.78
#> AIC: 21355.55
```

```
#>
#> Location coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  2.32064    0.06051  38.350   <2e-16 ***
#> sexmale     -0.22457    0.08723  -2.574     0.01 *
#>
#> Scale coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  1.10279    0.01823  60.497  <2e-16 ***
#> sexmale      0.01709    0.02618   0.653   0.514
#>
#> Lower threshold coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)     9.88      28.40   0.348    0.728
#>
#> Upper threshold coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  0.27743    0.09449   2.936  0.00332 **
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We find the following (for the UK in February and May 2009):

1. The intercept for the lower threshold is large but not significantly different from 0, meaning that $\hat{\gamma}_l$ is close to -1 but we cannot reject H_0: $\gamma_l = -.5$.
2. The intercept for the upper threshold is significantly larger than 0, meaning that we reject H_0: $\gamma_u = .5$ in favor of H_1: $\gamma_u > .5$.

Since $\hat{\gamma}_l \approx -1$, setting $\bar{\gamma}_l := -1$ may be restrictive. So we set $\bar{\gamma}_l := -2$ and reestimate the model.

```
lout <- hintreg(
  q2 ~ sex,
     ~ sex,
     ~ 1,
     ~ 1,
```

```
  data        = ias2009febmay,
  start       = vstart,
  threshbelow = NULL,
  threshabove = 1:5,
  limenlb     = -2,
  limenub     = 1
)
summary(lout)
#> Call:
#> hintreg(location = q2 ~ sex, scale = ~sex, lower = ~1, upper = ~1,
#>     data = ias2009febmay, start = vstart, threshbelow = NULL,
#>     threshabove = 1:5, limenlb = -2, limenub = 1)
#>
#> No. of iterations: 48
#> log-likelihood: -10670.38
#> AIC: 21352.76
#>
#> Location coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  2.29761    0.06313  36.396   <2e-16 ***
#> sexmale     -0.22822    0.08877  -2.571   0.0101 *
#>
#> Scale coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  1.11937    0.02085  53.687   <2e-16 ***
#> sexmale      0.01801    0.02620   0.688    0.492
#>
#> Lower threshold coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)   0.2377     0.1471   1.616    0.106
#>
#> Upper threshold coefficients:
#>             Estimate Std. Error t value Pr(>|t|)
#> (Intercept)   0.1843     0.1137   1.622    0.105
```

Now we find the following (for the UK in February and May 2009):

1. The intercept for the lower threshold is positive but not significantly dif-

ferent from 0, meaning that $\hat{\gamma}_l < -1$ but we cannot reject H_0: $\gamma_l = -1$.

2. The intercept for the upper threshold is not significantly different from 0, meaning that we can no longer reject H_0: $\gamma_u = .5$.

5 Heterogeneous normal interval regression model

Finally, we introduce heterogeneity into the indifference limen. Here we set the starting values as follows:

mean equation: use the estimated coefficients for the mean equation in a generalized normal interval regression model with an indifference limen

standard deviation equation: use the estimated coefficients for the standard deviation equation in a generalized normal interval regression model with an indifference limen

threshold equations: use the estimated thresholds in a generalized normal interval regression model with an indifference limen as the intercepts; set the other coefficients to 0

One can implement this procedure as follows:

```
vstart_beta  <- lout$coefL
vstart_delta <- lout$coefS
ck           <- length(vstart_beta)
vstart_lower <- c(lout$coefLower, numeric(ck - 1))
vstart_upper <- c(lout$coefUpper, numeric(ck - 1))
vstart       <- c(vstart_beta, vstart_delta, vstart_lower, vstart_upper)
```

Since $\hat{\gamma}_l < -1$ in the previous result, we set $\bar{\gamma}_l := -2$, and fit a heterogeneous normal interval regression model to interval data with an indifference limen as follows:

```
hintreg(
  q2 ~ sex,
     ~ sex,
```

```
    ~ sex,
    ~ sex,
  data        = ias2009febmay,
  start       = vstart,
  threshbelow = NULL,
  threshabove = 1:5,
  limenlb     = -2,
  limenub     = 1
) |> summary()
#> Call:
#> hintreg(location = q2 ~ sex, scale = ~sex, lower = ~sex, upper = ~sex,
#>     data = ias2009febmay, start = vstart, threshbelow = NULL,
#>     threshabove = 1:5, limenlb = -2, limenub = 1)
#>
#> No. of iterations: 47
#> log-likelihood: -10668.39
#> AIC: 21352.77
#>
#> Location coefficients:
#>              Estimate Std. Error t value Pr(>|t|)
#> (Intercept)  2.29054    0.06515  35.155   <2e-16 ***
#> sexmale     -0.21248    0.09398  -2.261   0.0238 *
#>
#> Scale coefficients:
#>              Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 1.126183   0.023248  48.441   <2e-16 ***
#> sexmale     0.003834   0.033486   0.114    0.909
#>
#> Lower threshold coefficients:
#>              Estimate Std. Error t value Pr(>|t|)
#> (Intercept)   0.3667     0.2154   1.702   0.0887 .
#> sexmale      -0.2539     0.2956  -0.859   0.3903
#>
#> Upper threshold coefficients:
#>              Estimate Std. Error t value Pr(>|t|)
#> (Intercept)   0.3124     0.1560   2.003   0.0452 *
#> sexmale      -0.2574     0.2283  -1.127   0.2595
#> ---
```

```
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We find the following (for the UK in February and May 2009):

1. The intercepts for the lower and upper thresholds are significantly larger than 0, meaning that we reject $H_0:\gamma_l=-1$ and $H_0:\gamma_u=.5$ in favor of $H_1:\gamma_l<-1$ and $H_1:\gamma_u>.5$, respectively, at least for women.
2. The indifference limens for men and women are not significantly different.

References

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