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Introduction

Rubinstein's (1982) model predicts (or describes) a unique outcome in an alternating-offer bargaining model. The result, unfortunately, is heavily dependent on the bargaining procedure. Multiple equilibria may be obtained by slight modification of the procedure; e.g., Fernandez and Glazer (1991) and Muthoo (1990). While these studies have contributed to bargaining theory by pointing out possibilities of multiple equilibria, they are unsatisfactory theories of bargaining per se for just that reason. In addition, some extreme allocations supported by such equilibria are not plausible outcomes. We should, therefore, view these results with skepticism.

In this paper we propose a way of restricting the set of multiple equilibria in bargaining models for which many equilibria exist, using complexity considerations as in Abreu and Rubinstein (1988), Piccione-Rubinstein (1993), and Rubinstein (1986). We consider a machine game as in Abreu and Rubinstein (1988) induced from a bargaining model with ratification, as in Muthoo (1990). Complexity considerations can be incorporated into the machine game. We assume that players are concerned not only with payoffs but also with the complexity of strategies used in the machine game.

We use a Moore machine (simply machine) to express a player's concern about the complexity of his/her strategy. This machine has been introduced into the analysis of repeated games in Neyman (1985), Rubinstein (1986), Abreu and Rubinstein (1988), Piccione and Rubinstein (1993) and many other papers. Although it is applicable in any strategic situations, there have been few applications except in the analysis of repeated games. A few exceptions are Osborne and Rubinstein (1990), Binmore et al. (1998) and Chatterjee and Sabourian (2000). Osborne and Rubinstein (1990) uses the language of machines to express players' strategies for convenience, but complexity of strategies does not enter the analysis. Binmore et al. (1998) and Chatterjee and Sabourian (2000) use the automaton in the machine game of alternating-offer bargaining.

There are two types of machines. Rubinstein (1986) and Abreu and Rubinstein (1988) use the "exact automaton": its input consists only of the opponent's previous action in an infinitely repeated game. Kalai and Stanford (1988) use the "full automaton": its input consists not only of the opponent's previous action but also its own previous action. The latter corresponds more closely to the game-theoretic concept of strategy. If we employ a sort of Nash equilibrium (which will be called automaton Nash equilibrium) as the equilibrium concept in the machine game, then either machine is appropriate. If, however, a sort of subgame perfect equilibrium (which will be called automaton subgame perfect equilibrium) is considered, the full automata are more appropriate.

We will show that many of the equilibria of Muthoo's model are eliminated by slight complexity restrictions. In this sense these eliminated equilibria can be considered as unstable with respect to complexity consideration. This is due to different structures among the subgame perfect equilibria in Muthoo's model. Almost all subgame perfect equilibria in Muthoo's bargaining model are formed from two-state strategies, and so are the automata that are induced by such strategies. The reason for elimination of such strategies is that in the machine game such automata have unused states on the equilibrium path, and hence cannot be used in subgame perfect equilibria when there are complexity considerations. That is, we can always find profitable deviations that use only one state on the purported subgame perfect equilibrium path in the machine game. We will show that only one-state machines are used in subgame perfect equilibrium of the machine game, at least when a somewhat restricted class of strategies is permitted, and only Muthoo's stationary equilibria survive such restrictions. Note that we do not assume that players use only stationary strategies a priori. The stationarity results from equilibrium considerations.

The paper is organized as follows. Section 2 specifies a bargaining model without commitments as in Muthoo (1990) and a machine game induced by the bargaining game. We will show first that some subgame-perfect equilibrium strategies in the bargaining game are not equilibria in the induced machine game. We will then characterize the general equilibrium structure of the machine game in section 3. Only stationary strategies are shown to survive. The last section concludes.

2　Bargaining Model and Machine Game

In this section a bargaining model and the corresponding machine game are defined. Then Muthoo's equilibrium structures will be examined.

2.1　Bargaining Model

We consider a bargaining model without commitment (Muthoo (1990)). Player 1 and player 2 bargain over an item (X) of size 1. We consider augmented

Rubinstein-style alternating-offer bargaining. Player 1 proposes an allocation of the good at the beginning of period 0. Player 2 responds by accepting or rejecting the offer. If player 2 rejects the offer (R), then the game goes to period 1. If player 2 accepts the offer (A), then player 1 accepts or rejects the acceptance of player 2. If player 1 accepts player 2's acceptance (AA), the game ends. If player 1 rejects (RA), then the game goes to period 1. In period 1 the roles of the players are reversed. In even periods player 1 begins by offering and in odd periods player 2 begins by offering. The bargaining continues until some offer is accepted and that acceptance is accepted. A period is indexed by $t=0, 1, 2, \ldots$. Within a period, there are at most three types of actions taken by players in sequence. Thus actual time is indexed by tr, where sub-period $r=1, 2, 3$.

The players have instantaneous von-Neumann-Morgenstern utility functions over the sizes of their shares of the item. We assume that the players consume the good immediately after agreement. The players' payoffs from the split $(x, 1-x)$ are $\delta^t x$ for player 1 and $\delta^t(1-x)$ for player 2, where t is the period when the players reach an agreement and $\delta \in [0, 1)$ is the players' common discount factor. Let this bargaining game be Γ.

The set of choices available to a player in the game is denoted by $C \equiv [0, 1] \cup \{A \cup R\} \cup \{AA \cup RA\}$. We shall denote a history of outcomes in a period by e, and the set of all such possible histories of an even period by E and that of an odd period by E'. A history of a period is a complete account of what happened in the bargaining in a given period.

Denote a partial history within a period by h and the set of such partial histories by H. Examples of $h \in H$ could be the null (empty) set $\emptyset$, an offer (x), an offer x followed by an acceptance (x, A), and so on. If h is the null history, the period is just beginning and an offer has yet to be made.

Denote the set of partial histories (the information sets) for player i in any period by H_i. Thus $H_i \equiv \{h \in H \mid \text{it is } i\text{'s turn to play after } h\}$. Then, denote the set of choices available to a player i, given a partial description $h_i \in H_i$, by $C_i(h_i)$:

$$C_i(h_i) = \begin{cases} [0, 1] & \text{if it is player } i\text{'s turn to propose an allocation after } h_i \\ \{A, R\} & \text{if it is player } i\text{'s turn to decide } A \text{ or } R \text{ after } h_i \\ \{AA, RA\} & \text{if it is player } i\text{'s turn to decide } AA \text{ or } RA \text{ after } h_i \end{cases}$$

Let E^t be the set of all possible histories of outcomes of t periods. Also let $E^\infty \equiv \{\emptyset\} \cup (\cup_{t=0}^{\infty} E^t)$ be the set of all possible finite histories of periods. Then a strategy for player i is a function $\sigma_i : E^\infty \times H_i \to C$, where $\sigma_i(\emptyset, e^0, \ldots, e^t, h_i) \in C_i(h_i)$ for any $(\emptyset, e^0, \ldots, e^t) \in \{\emptyset\} \times E^t$ and for any partial history $h_i \in H_i$.

We need extra notation. $p_i(e^\infty)$ is the payoff obtained by player i on reaching $e^\infty \in E^\infty$. $e^\infty(\sigma)$ is the end-node which is reached when $\sigma \in \Sigma$ is played. Denote the period at which the players reach agreement when they play σ_1 and σ_2 by $t(\sigma_1, \sigma_2)$. $t(\sigma_1, \sigma_2)$ is either finite or infinite depending on the strategy combination $\sigma = (\sigma_1, \sigma_2)$. Denote the subgame of the bargaining game after a history $(e^1, \ldots, e^t, h)$ by $\Gamma(e^1, \ldots, e^t, h)$.

2.2　Machine Games

We construct a machine game induced from the above bargaining game Γ. We employ an automaton (machine) of the type considered by Osborne and Rubinstein (1990) instead of a standard automaton as considered by Hopcroft and Ullman (1979). In our machine an action is prescribed by an output function which depends not only on the state but also on publicly known variables. The publicly known variables include the identity of the player whose turn it is to move, the type of action the player has to take, and (possibly) the amount of an offer. We distinguish the state variables from the publicly known variables. All information available from the game tree is excluded from the state description. That is, our automata need to know much more information about the game tree than standard automata. Using an automaton of the type of Osborne and Rubinstein (1990) has advantages in clarifying the structure of the strategies in the bargaining game Γ. We will discuss this when a stationary machine is defined.

The set of strategies is restricted to those strategies that can be played by a finite automaton. Finite automata have been adopted extensively in analysis of repeated games. To introduce finite automata that implement strategies in the bargaining game, we consider the following machines.

Definition 1. The Machine is $M_i=(Q_i, q_i^0, f_i, \tau_i)$, where

- Q_i is a finite set of states,
- q_i^0 is the initial state that belongs to Q_i,
- $f_i : Q_i \times H \rightarrow C$ is the output function mapping states and partial histories before player i's move within a period into the set of action,

$$f_i(q_i, h_i) \in C_i(h_i),\ \forall q_i \in Q_i \text{ and } \forall h_i \in H_i,$$

- $\tau_i : Q_i \times H \rightarrow Q_i$ is a transition function mapping the current states and partial histories within a period into the set of states.

If the machine has only a single state, the output function f_i must specify the same action whenever the same partial histories that belong to H_i have occurred. Thus in order to specify two different actions under the same $h_i \in H_i$, the machine needs at least two states. The transition function specifies how the machine changes its action. Notice that while the output function is active only when the player using the automaton can move, the transition function is always

Table 1: Automation in Rubinstein Model

		state q^0
M_1	demands	$\frac{1}{1+\delta}$
	accepts	$x \geq \frac{\delta}{1+\delta}$
M_2	proposes	$\frac{\delta}{1+\delta}$
	accepts	$x \leq \frac{1}{1+\delta}$

active so that the machine can move to another state anytime.(1)

For example, the equilibrium strategy in the Rubinstein model (Rubinstein (1982) can be played by the one-state machines (M_1, M_2) in Table 1:

Player 1's machine is $M_1=(Q_1, q^0, f_1, \tau_1)$ such that(2)

- $Q_1=\{q^0\}$

- $$f_1(q^0, h_1)=\begin{cases} \frac{1}{1+\delta} & \text{if } h_1 \in H_1 \text{ is such that player 1 is the proposer} \\ A & \text{if } h_1 \in H_1 \text{ is such that player 1 is a responder and} \\ & x \geq \frac{\delta}{1+\delta} \text{ is proposed during } h_1 \\ R & \text{if } h_1 \in H_1 \text{ is such that player 1 is a responder and} \\ & x < \frac{\delta}{1+\delta} \text{ is proposed during } h_1 \end{cases}$$

- $\tau_1(q^0, h)=q^0$ for all $h \in H$.

(1) Here we have not defined the machine rigorously in terms of its implementability because our machines are assumed to have much more abilities to implement strategies than the standard automata in Hopcroftand Ullman (1979). We have simply assumed that the automaton can know enough information to determine actions. Our automata are just convenient expressions for strategies. We will discuss this problem later.

(2) Notice that there are no nodes where a player decides to whether or not retract his/her offer in Rubinstein's model.

Similarly we can specify the machine M_2 for player 2.

Once a machine is defined, we can construct the corresponding strategy in the bargaining game Γ from the machine. $f_i(q_i(\emptyset, e^0, \dots, e^t), h_i)$, $\forall(\emptyset, e^0, \dots, e^t) \in \{\emptyset\} \times E^t$ and $\forall h_i \in H_i$ is called the Γ-strategy induced by the machine M_i, where $q_i(\emptyset, e^0, \dots, e^t)$ is the state of the machine after a history $(\emptyset, e^0, \dots, e^t)$. After the history $(\emptyset, e^0, \dots, e^t)$, the prevailing state is determined by the transition function $\tau_i : Q_i \times H \rightarrow Q_i$. Denote such a Γ-strategy induced by the machine M_i by $\sigma_i(M_i)$. The combination of machines (M_1, M_2) induces the pure strategy combination $(\sigma_1(M_1), \sigma_2(M_2))$ in the bargaining game.

Given that $M = (Q, q^0, f, \tau)$, let $M(q)$ be the machine such that

$$M(q) = (Q, q, f, \tau),\ q \in Q.$$

That is, M and $M(q)$ differ in only the initial states.

By a stationary machine we mean an essentially one-state automaton. An essentially one-state machine is defined by $M = (Q, q^0, f, \tau)$ such that for each $h_i \in H_i$, $f_i(q, h_i) = f_i(q', h_i)$, $\forall q, q' \in Q$. If M has only one state, M is an essentially one-state machine. Essentially one-state machines, however, may have more than one state if the output functions specify the same actions for any $q \in Q$. We call an essentially one-state machine a stationary machine because the Γ-strategy induced by the essentially one-state machine is a stationary strategy in the bargaining game Γ no matter what strategy the opponent uses. For example, the machine in Table 1 is a one-state machine that induces a stationary strategy in Rubinstein's bargaining model.

Define a bargaining machine game in the following way. Each player chooses a machine from $\mathcal{M}_i$, where $\mathcal{M}_i$ is the set of all finite automata in our sense. The machines chosen by the players play the game for the players. Each machine starts from its initial state. Each machine chooses an action whenever it can choose.[3] Then the machine's state changes by the transition rule. Then each machine chooses an action given the new state. The machines stop only after the machines' joint play leads to an agreement. If they do not ever reach an agreement, the machines do not stop.[4]

Player i's payoff $\pi_i(M_1, M_2)$ in the machine game is represented by the payoff $\delta^{t(\sigma(M))}p_i(e^{\infty}(\sigma(M)))$ in the bargaining game, where $\sigma(M)$ is the strategy combination induced by the combination of machines $M=(M_1, M_2)$. Let $c(M_i)$ be the complexity of a machine M_i. The complexity is measured by the number of states in M_i. Each player has a preference of $(M_1, M_2)\succ_i(L_1, L_2)$ whenever (1) $\pi_i(M_1, M_2)>\pi_i(L_1, L_2)$ or (2) $\pi_i(M_1, M_2)=\pi_i(L_1, L_2)$ and $c(M_i)<c(L_i)$. Thus we assume that players have lexicographic preference over allocations of the good and complexity of the machine (see Rubinstein (1986)).

We define two equilibrium concepts for machine games as follows.

Definition 2. (Automaton Nash equilibrium (ANE)). The pair of machines (M_1^*, M_2^*) is an automaton Nash equilibrium if there is no i and $M_i'\in\mathcal{M}_i$ such that

$$(M_i', M_j^*)\succ_i(M_i^*, M_j^*).$$

In our machine games induced from the bargaining game Γ, there exists an ANE.

Proposition 1. *There exists an ANE* (M_1^*, M_2^*) *in the machine game induced*

(3) Notice that since the bargaining game is an extensive form game, each machine may have to wait to choose an action until the rule of the game prescribe that it does so.

(4) Thus the machines can know where they are on the game tree and can recognize the role of each node.

Table 2: ANE Machines

		state q^0
	demands	$\bar{x}$
M_1	accepts	$x \geq \bar{x}$
	accepts the acceptance	$x \geq \delta\bar{x}$
	proposes	$\bar{x}$
M_2	accepts	$x \leq \bar{x}$
	accepts the acceptance	$(1-x) \geq \delta\bar{x}$
transitions		Absorbing

Note: $\bar{x} \in [0, 1]$. "Absorbing" means $\tau_i(q^0, h) = q^0$ for all $h \in H$.

from the bargaining game Γ.

Proof. Consider the machines in Table 2. These machines constitute an ANE: We first check player 1's optimality. Suppose that M_2 is fixed. If M_1 is played, then (M_1, M_2) generates a payoff $\bar{x}$ for player 1 at period 0. Consider player 1's deviation. Firstly, suppose that players can choose machines with at most one state. If M_1 demands $x < \bar{x}$, then the proposal is accepted. Player 1 gets $x < \bar{x}$, which is worse. If M_1 demands $x > \bar{x}$, then the proposal is rejected. M_2 proposes $\bar{x}$ at period 1. If M_1 does not change the acceptance rule, player 1 gets $\delta\bar{x}$ at period 1. If M_1 changes the acceptance rule, M_1 rejects $\delta\bar{x}$. Player 1 gets at most $\bar{x}$ at period 2. If M_1 changes only the acceptance of acceptance rule (rejects her own offer), player 1 gets $\bar{x}$ at period 1 or $\bar{x}$ at period 2. Thus any deviations using a single state are not profitable. Similarly, we can check the optimality of M_2 relative to single-state deviating machines.

Next consider deviations by machines with two states. Suppose that M_2 is fixed. Using any two-state machine, player 1 can get at most $\bar{x}$ at period $t = 0, 2, \ldots$ when M_1 demands an amount of the good (and M_1 accepts player 2's acceptance), or $\delta\bar{x}$ at period $t = 1, 3, \ldots$ when M_2 proposes an allocation of the good.

Thus we cannot find machines that generate more than $\bar{x}$ for player 1. Therefore the machine M_1 in Table 2 is a best response to the machine M_2 in Table 2. Similarly, we can check the optimality of M_2 relative to two-state deviating machines.

The argument is similar for deviations by machines with more than two states.

□

There is so far no consideration of the dynamics of bargaining in the ANE of the machine game. The following solution concept (Neme and Quintas (1995)) incorporates such dynamics.

Definition 3. (Automaton subgame perfect equilibrium (ASPE)). The pair of machines (M_1^*, M_2^*) is an automaton subgame perfect equilibrium if after every history in the bargaining game Γ, the pair of machines (M_1^*, M_2^*) is an ANE in the machine game where the machine game begins from the prevailing state. That is, (M_1^*, M_2^*) is an automaton subgame perfect equilibrium if for all $(e^1, \ldots, e^t, h_i) \in E^t \times H_i$, there is no M_i' such that

$$(M_i', M_j^*(q_j(\emptyset, e^0, \ldots, e^t, h_i)))$$
$$\succ_i (M_i^*(q_i(\emptyset, e^0, \ldots, e^t, h_i)), M_j^*(q_j(\emptyset, e^0, \ldots, e^t, h_i))).$$

where $q_j(\emptyset, e^0, \ldots, e^t, h_i)$ is the state after h_i in period t.

Although the machines in Table 2 constitute an ANE, if $\bar{x} \neq 1/(1+\delta)$, then the machines do not constitute an ASPE. There exist profitable deviations at some subgames. For example, there is a profitable deviation by player 2 at time 02 after M_1 demands an amount of the good x such that $(1-\bar{x}) > (1-x) > \delta(1-\bar{x})$. Such an x will be rejected if player 2 uses M_2 in Table 2. But player 2 should accept such an offer x because after the rejection, player 2 can get at most the present value $\delta(1-\bar{x}) < 1-x$.

It might seem that if strategies constitute Nash equilibria or subgame perfect equilibria in the bargaining game Γ, then the corresponding machines constitute ANE or ASPE in the corresponding machine game. That, however, is not true. As we will show in the next section, some machines that induce subgame perfect equilibrium strategies in Muthoo's (1990) model are not even ANE in the corresponding machine game.

We will study the following issues in subsequent sections.

(1) Do all the equilibrium strategies of Muthoo (1990) survive with consideration of complexity?

Muthoo (1990) showed that all possible allocations can be supported as subgame perfect equilibrium. Folk-theorem-like strategies are used to support such equilibria. We will show that there are subgame perfect equilibrium paths of Γ that are not generated by any ANE (or ASPE) machine pairs (M_1^*, M_2^*).

(2) What are the ANE and ASPE in Muthoo's model?

We will show that the Muthoo model is (strategically) equivalent to the Rubinstein model when complexity consideration are taken into account. That is, both the Rubinstein model and the Muthoo model with complexity consideration generate the same equilibrium outcomes using similar stationary strategies.

2.3 Equilibria in Muthoo's Model

In this section we show that some equilibrium strategies in Muthoo's model are not ANE.

Muthoo showed that the (induced) stationary strategies in Table 3 constitute a subgame perfect equilibrium in the bargaining game. This pair of machines (M_1, M_2) is an ANE because, given M_2, player 1 cannot get a higher payoff than $1/(1+\delta)$, and adding a state just generates a cost of implementing the machine without leading to a higher payoff than $1/(1+\delta)$. Similarly there is no profitable

Table 3: Muthoo

		state q^0
M_1	demands	$\frac{1}{1+\delta}$
	accepts	$x \geq \frac{\delta}{1+\delta}$
	accepts the acceptance	$x \geq \frac{\delta^2}{1+\delta}$
M_2	proposes	$\frac{\delta}{1+\delta}$
	accepts	$x \leq \frac{\delta}{1+\delta}$
	accepts the acceptance	$(1-x) \geq \frac{\delta^2}{1+\delta}$

deviation for player 2. Thus those stationary equilibrium strategies in the bargaining game are represented by the ANE machines in the corresponding machine game.

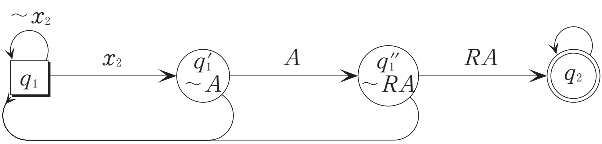
Muthoo (1990) also showed that there is a subgame perfect equilibrium $(x, t)=(1, 0)$, where (x, t) means that the players reach the agreement x for player 1's share at period t. Such equilibrium strategies have different structure from the strategies expressed in Table 3. The equilibrium strategies that generate the extreme allocation specify two different actions at the same h_i. Thus from the definition of our output function, in order to express such an equilibrium strategy by a machine, it needs at least two states. The pair of essentially two-state machines (M_1, M_2) that generate this subgame perfect equilibrium are summarized in Table 4. Notice here we use four states to express the equilibrium strategies by the machines. But under states q_1, q_1', and q_1'', the output functions specify the same actions. Thus we call such machines essentially two-state machines. All equilibrium strategies in Muthoo (1990) except the strategies of

Table 4:

		state q_1, q_1', q_1''	state q_2
M_1	demands	1	$\frac{1}{1+\delta}$
	accepts	$x \geq 1$	$x \geq \frac{\delta}{1+\delta}$
	accepts the acceptance	$x \geq \delta$	$x \geq \frac{\delta^2}{1+\delta}$
M_2	proposes	1	$\frac{1}{1+\delta}$
	accepts	$x \geq 0$	$x \leq \frac{\delta}{1+\delta}$
	accepts the acceptance	$x = 1$	$(1-x) \geq \frac{\delta^2}{1+\delta}$

Note: $\delta > 1/\sqrt{2}$. The transitions occur as in Figure 1.

Figure 1: Transition Diagram for the Muthoo High Machine



Note: $x_2 \in [\delta, 1)$. $\sim \cdot$ means that $\cdot$ dose not occur.

Table 3 need more than one state to be expressed by machines because such equilibrium strategies specify different actions at some same h_i.

The initial states for both machines in Table 4 are q_1. The state q_2 is reached from q_1 when the following history occurs (see Figure 1): Player 2 has offered $x_2 \in [\delta, 1)$ (the state moves to q_1'), player 1 has accepted the offer (the state moves to q_1''), and then player 2 has rejected player 1's acceptance. When the above history occurs, the machine reaches absorbing state q_2. Notice that in the machine game both players have the same set of states, and in state q_2 the output functions are the same as in Table 3.

In the bargaining game, $(\sigma_1(M_1), \sigma_2(M_2))$ is a subgame perfect equilibrium for sufficiently high δ. Player 2, for instance, cannot find profitable deviations in any subgames. Suppose that player 2 deviates to get a positive payoff, proposing $x_2 \in [\delta, 1)$. It seems that player 1 would accept such an offer and player 2 would accept player 1's acceptance. But after player 1 accepted such an offer, it is optimal for player 2 to reject player 1's acceptance for sufficiently high discount factors because both players change the state to q_2 after player 2's rejection, and so player 2 can get the present value $\delta(\delta/(1+\delta))$ which is higher than $1-x_2$ for $x_2 \in [\delta, 1)$ and $\delta > 1/\sqrt{2}$.

Is (M_1, M_2) an ANE? We claim that (M_1, M_2) is not even an ANE in the machine game, even though the strategies induced from (M_1, M_2) are subgame perfect equilibrium in the bargaining game. Notice that on the equilibrium path, state q_2 is not used. Players using machines M_1 and M_2 in Table 4 reach agreement at period 0. Let M_2 be given. Of course, player 1 cannot get a higher payoff than 1. Player 1, however, can eliminate state q_2 keeping the same payoff of 1 because q_2 is not used on the equilibrium path. By using the stationary machine which is used under q_1, player 1 can get the same payoff. Thus there is a profitable deviation for player 1 so that (M_1, M_2) is not an ANE. We conclude that at least this (M_1, M_2) cannot support $(x, t)=(1, 0)$ as the equilibrium outcome in the machine game. At this point there may or may not exist other automata (M_1, M_2) that achieve $(x, t)=(1, 0)$. No subgame perfect equilibria constructed by Muthoo, however, are ANE in the corresponding machine game except the strategies induced by the stationary machines in Table 3. This is because all such eliminated strategies have unused states on the equilibrium path.

From the above example, it seems therefore that ANE requires that all states be used on the equilibrium path. If so, from the definition of ASPE, punishments to prevent deviations must be executed with actions on the equilibrium path when deviations occur, because all states of machines must be used.

3 Structure of Automata Equilibrium

In this section we characterize general properties of ANE and ASPE in the machine game induced from the bargaining game without commitment. Some results from the repeated game literature are applied to our model in the arguments.

Lemma 1. *If (M_1^*, M_2^*) is an ANE of a machine game, then for every state q_i of the machine M_i^* there exists a partial history h such that $q_i(\emptyset, e^0, \ldots, q^t, h) = q_i$, where $q_i(\emptyset, e^0, \ldots, q^t, h)$ is the realized state after $(\emptyset, e^0, \ldots, q^t, h)$.*

Proof. Otherwise, player i can eliminate a state which is not used in equilibrium keeping the same payoff. This is a contradiction to the fact that (M_1^*, M_2^*) is an ANE. □

Lemma 1 shows that all states of equilibrium machines must be used on the equilibrium path. From this lemma the multiple-state machines of Muthoo (1990) are easily proved not to be ANE machines because only one state is used on the equilibrium path by each of them.

Let M_j be fixed. We show that there exists M_i with the same set of states and transition function as M_j, the induced strategy of which achieves the highest payoff in the bargaining game Γ for i against Γ-strategy $\sigma_j(M_j)$.

Lemma 2. *For any $M_j \in \mathcal{M}_j$, there exists $M_i \in \mathcal{M}_i$ that uses the same set of states,* *the same initial state, and the same transition function as* M_j *such that* $\pi_i(M_i, M_j) \geq \pi_i(M_i', M_j)$ *for any* $M_i' \in \mathcal{M}_i$.

Proof. Remember that $\pi_i(M_i, M_j) = \delta^{t(\sigma(M))} p_i(e^{\infty}(\sigma(M)))$. Given a machine $M_j = (Q_j, q_j^0, f_j, \tau_j)$, consider the Γ-strategy $\sigma_j(M_j)$ for player j. In the bargaining game Γ, let $\sigma_i^* | \sigma_j(M_j)$ be such that

$$\sigma_i^* | \sigma_j(M_j) \in \operatorname{argmax}_{\sigma_i} \delta^{t(\sigma_i, \sigma_j(M_j))} p_i(e^{\infty}(\sigma_i, \sigma_j(M_j))).$$

So $\sigma_i^* | \sigma_j(M_j)$ is a best response for player i to $\sigma_j(M_j)$. Then we have the history that results in the end point $e^{\infty}(\sigma_i^* | \sigma_j(M_j), \sigma_j(M_j))$ when $\sigma_i^* | \sigma_j(M_j)$ for player i and $\sigma_j(M_j)$ for player j are played. Consider a machine $M_i^* = (Q_j, q_j^0, f_i^*, \tau_j)$. The machine M_i^* has the same set of states, initial state, and transition function as M_j. Therefore, $c(M_j^*) = c(M_j)$. f_i^* will be defined below. We investigate if there exists M_i^* that can induce the best response $\sigma_i^* | \sigma_j(M_j)$ given $\sigma_j(M_j)$. In order to show this, it is enough to find a Γ-strategy $\sigma(M_i^*)$ such that $(\sigma_i(M_i^*), \sigma_j(M_j))$ generates the history that results in $e^{\infty}(\sigma_i^* | \sigma_j(M_j), \sigma_j(M_j))$.

Step 1. First suppose that M_j uses only one state (the initial state) on the path leading to the end point $e^{\infty}(\sigma_i^* | \sigma_j(M_j), \sigma_j(M_j))$. When players reach agreement at period 0 or 1, it is sure that there exists M_i^* with $\sigma_i(M_i) = \sigma_i^* | \sigma_j(M_j)$ because M_j does not change its actions on the path, and f_i^* can simply specify actions that lead to the agreement at period 0 or 1. Now if they reach agreement at period $t \geq 2$, they will also reach agreement at period $t-2$ because $\sigma_j(M_j)$ specifies the same actions at period $t-2$ and t. So when M_j is given, it is enough for M_i^* to use only one state to reach the end point $e^{\infty}(\sigma_i(M_i^*), \sigma_j(M_i)) = e^{\infty}(\sigma_i^* | \sigma_j(M_j), \sigma_j(M_j))$. Such M_i^*, however, may have the same number of states (>1) as M_j, but uses only one on the path.

Step 2. Suppose that M_j uses two states q_1 and q_2 on the path leading to the end point $e^{\infty}(\sigma_i^*|\sigma_j(M_j), \sigma_j(M_j))$, and the output function $f_j(q_1, h_j) \neq f_j(q_2, h_j)$ for some h_j. Suppose that state q_2 is reached at time $t'r'$ and players reach agreement under the new state.(5) Because M_i^* has the same set of states as M_j, a pair of machines (M_i^*, M_j) can reach q_2 by some actions that can be specified by an M_i^*. That is, at the initial state q_i, M_i^* can specify actions that trigger a state change, and then can choose any optimal actions under the new state. As in Step 1, M_i^* may have more than two states, though it is enough for M_i^* to use only two states to reach the end point $e^{\infty}(\sigma_i^*|\sigma_j(M_j), \sigma_j(M_j))$.

Step 3. Similarly we can show that if M_j uses n states under which f_j specifies different actions with some same h_j on the path leading to the end point $e^{\infty}(\sigma_i^*|\sigma_j(M_j), \sigma_j(M_j))$, the pair of machines (M_i^*, M_j) can reach the end point for some M_i^* that uses at least n states. This process ends in finite times because M_j has only a finite number of states. Thus we can always find $M_i^* = (Q_j, q_j^0, f_i^*, \tau_j)$ such that $\pi_i(M_i^*, M_j) \geq \pi_i(M_i', M_j)$ for any M_i'. □

Lemma 3. *For any ANE* (M_1^*, M_2^*), $c(M_1^*) = c(M_2^*)$.

Proof. From Lemma 2, for any M_2^*, we can find M_1 such that $c(M_1) = c(M_2^*)$ and $\pi_1(M_1, M_2^*) \geq \pi_1(M_1', M_2^*)$ for any $M_1' \in \mathcal{M}_1$. Thus in ANE, $c(M_1^*) \leq c(M_1) = c(M_2^*)$. Similarly, $c(M_2^*) \leq c(M_2) = c(M_1^*)$. Therefore $c(M_1^*) = c(M_2^*)$. □

Lemma 3 asserts that the number of states of equilibrium machines are the same. That implies that the equilibrium machines have the same set of states because the characteristics of states of one machine do not affect those of states of

(5) If the two states q_1 and q_2 are used more than once on the path, we have to repeat the following argument.

the other machine. Lemma 2 says that given any player j's machine, we can find player i's machine with the same set of states, the same initial state and the same transition function as player j's machine that can achieve the highest payoff. Although it is possible for M_i with fewer states than given M_j^* to achieve the highest payoff, the equilibrium machines have the same set of states from the above argument. Thus equilibrium machines M_1^* and M_2^* must use the same set of states, the same initial state and the same transition function.

Proposition 2. *Only stationary machines can be ANE in the machine game constructed from the bargaining model without commitment. The strategy pairs induced by such stationary ASPE machine pairs generate the Rubinstein equilibrium outcome.*

Proof. Let (M_1^*, M_2^*) be an ASPE. Suppose that both machines have essentially two states under which the transition functions specify different actions: From Lemma 3, we can let $Q_i = \{q_1, q_2\}$ for both i. Both M_1^* and M_2^* also have the same transition function. W.l.o.g., suppose $q_1^0 = q_1$. Because M_i^* is an equilibrium machine, both q_1 and q_2 should be used in the equilibrium from Lemma 1. Suppose that the game reaches the end-point under q_2.(6) We show that in this case we can always find a profitable deviation.

Subgames after q_2 occurs: Let $(\emptyset, e^0, \ldots, e^t)$ be the equilibrium path of play. Because q_2 is the last state on the equilibrium path, there exists some partial history h such that after $(\emptyset, e^0, \ldots, h)$, the state goes to q_2 and stays there. Denote the time after $(\emptyset, e^0, \ldots, h)$ be $\bar{tr}$. ASPE requires that after every history in the

(6) The case where the game reaches the end-point under q_1 is handled similarly.

Table 5: Machines $M_1^*(q_2)$ and $M_2^*(q_2)$

		state q_2
$M_1^*(q_2)$	demands	$\frac{1}{1+\delta}$
	accepts	$x \geq \frac{\delta}{1+\delta}$
	accepts the acceptance	$x \geq \frac{\delta^2}{1+\delta}$
$M_2^*(q_2)$	proposes	$\frac{\delta}{1+\delta}$
	accepts	$x \leq \frac{1}{1+\delta}$
	accepts the acceptance	$(1-x) \geq \frac{\delta^2}{1+\delta}$

subgame $\Gamma(\emptyset, e^0, \dots, h)$, the pair of machines be an ANE of the machine game starting at q_2. We will show that the machines in Table 5 must be part of any ASPE machines.

If the machines in Table 5 play the game from the beginning of the period, that is from $\bar{t}1$, then previous offers under q_1 do not enter the analysis in this subgame and all responses are best. However, since we have supposed that there are essentially two states, the machines in Table 5 may begin playing after some behavior under q_1. If the state changes to q_2 after player i's offer at time $\bar{t}1$, then the effective offer $\bar{x}$ (player i's offer at time $\bar{t}1$) is not, in general, the same as $1/(1+\delta)$ or $\delta/(1+\delta)$ because we have not yet specified the actions that occur under q_1. Thus we need to check the subgame perfectness of the machines in Table 5 whatever happened under q_1.

First, consider the subgame beginning with player 1's offer. Let $M_2^*(q_2)$ be given. If player 1 follows $M_1^*(q_2)$, the outcome is $(1/(1+\delta), \bar{t})$. This holds for whatever the previous offer is under q_1 because we are now considering the subgames beginning with player 1's offer. When player 1 deviates after $(\emptyset, e^0, \ldots, h_1)$, then the outcome must be either (x, t) where $x \leq 1/(1+\delta)$ and $t \geq \bar{t}$, $(\delta/(1+\delta), t)$ where $t \geq \bar{t}+1$, or disagreement. Thus player 1 cannot get a higher payoff than $1/(1+\delta)$ by any deviation so that player 1's machine is the best response to player 2's machine in this subgame. Thus $M_1^*(q_2)$ is the best response to $M_2^*(q_2)$ in this subgame.

Next consider the subgames beginning with player 2's acceptance decision of an offer, $\Gamma(\emptyset, e^0, \ldots, h_2)$. Let $M_1^*(q_2)$ be given. If player 2 follows $M_2^*(q_2)$, then the outcome is $(1/(1+\delta), \bar{t})$ or $(\delta/(1+\delta), \bar{t}+1)$. Suppose that the outcome is $(1/(1+\delta), \bar{t})$ when the players use $(M_1^*(q_2), M_2^*(q_2))$. If player 2 deviates (that is, changes the criterion of either acceptance or rejection of player 1's offer), the outcome must be either $(1/(1+\delta), t)$ where $t \geq \bar{t}$, $(\delta/(1+\delta), t)$ where $t \geq \bar{t}+1$, or disagreement. Compare $(1/(1+\delta), \bar{t})$ and $(\delta/(1+\delta), \bar{t}+1)$. From

$$\delta^{\bar{t}}\left(1-\frac{1}{1+\delta}\right)-\delta^{\bar{t}+1}\left(1-\frac{\delta}{1+\delta}\right)=0,$$

player 2's machine is the best response to player 1's machine in this subgame. Next suppose that the outcome is $(\delta/(1+\delta), \bar{t}+1)$. Thus $\bar{x} > 1/(1+\delta)$, otherwise player 2 would have accepted the offer at period $\bar{t}$. When player 2 deviates, the outcome must be either $(\bar{x}, \bar{t})$, $(\delta/(1+\delta), \bar{t}+1)$ or disagreement. Because

$$\delta^{\bar{t}+1}\left(1-\frac{\delta}{1+\delta}\right)-\delta^{\bar{t}}(1-\bar{x})=\delta^{\bar{t}}\left(-\frac{1}{1+\delta}+\bar{x}\right)>0,$$

player 2's machine is the best response. Noting that we do not have to consider a deviation with more than one state, $M_2^*(q_2)$ is the best response to $M_1^*(q_2)$ in this subgame.

Finally consider the subgames beginning from $\bar{t}3$. Let $M_2^*(q_2)$ be given. If player 1 follows $M_1^*(q_2)$, the outcome is either $(1/(1+\delta), \bar{t})$ or $(\delta/(1+\delta), \bar{t}+1)$. Suppose that the outcome is $(1/(1+\delta), \bar{t})$. If player 1 deviates (that is, changes the criterion of either acceptance or rejection of player 2's acceptance of player 1's offer), the outcome must be either (x, t) where $x \leq 1/(1+\delta)$ and $t > \bar{t}$, or $(\delta/(1+\delta), t)$ and $t \geq \bar{t}+1$. Thus there is no profitable deviation for player 1. Next suppose that the outcome is $(\delta/(1+\delta), \bar{t}+1)$. Then $\bar{x} < \delta^2/(1+\delta)$. When player 1 deviates, the outcome must be either $(\bar{x}, \bar{t})$, $(\delta/(1+\delta), \bar{t}+1)$ or disagreement. Because

$$\delta^{\bar{t}+1}\left(\frac{\delta}{1+\delta}\right) - \delta^{\bar{t}}(\bar{x}) = \delta^{\bar{t}}\left(\frac{\delta^2}{1+\delta} - \bar{x}\right) > 0,$$

there is no profitable deviation. Thus player 1's machine is the best response to player 2's machine in this subgame. Therefore $M_1^*(q_2)$ is the best response to $M_2^*(q_2)$. Replacing the role of player 1 by player 2 and repeating the above argument, we can show that $(M_1^*(q_2), M_2^*(q_2))$ is an ASPE under q_2.

We have concluded that players use stationary machines from $\bar{t}\bar{r}$ on. Other induced machines may be ASPE; that is, the machines may go back to q_1 off the equilibrium path. Such machines, however, can be eliminated because the induced stationary machines generate the same payoff.

Uniqueness: Next let us show that the equilibrium $(M_1^*(q_2), M_2^*(q_2))$ is the unique ASPE at the subgame beginning at $\bar{t}\bar{r}$. Consider the machines in Table 2, where $\bar{x} \in [0, 1]$. The machines in Table 2 do not constitute an ASPE except for $\bar{x} = 1/(1+\delta)$ because we can find profitable deviations as discussed earlier.(7) From the previous result, we already know that only stationary machines consti-

(7) Notice that this discussion is different from the proof of uniqueness for the Rubinstein model, as in Shaked and Sutton (1984).

tute ANE in the subgame after q_2 occurs. Thus it is enough to pick out ASPE machines from such stationary ANE machines. Thus only machines in Table 5 constitute ASPE under q_2.

Whole game: Let us go back to the original problem. We have supposed that the bargaining reaches an agreement under q_2. Here we consider the simplest case where q_1 is used first, and then q_2 where the machine game ends.(8) We claim that ASPE requires that from the beginning of the bargaining game, players use the machines in Table 5. Consider the optimality of M_i^*. Suppose that the state change occurs at period t and t is an even number. Thus player 1 is the first mover at period t. (The case where t is an odd number is handled similarly.) We check when ASPE machines should move to state q_2. Remember that the pair of ASPE machines uses the same transition function.

Case 1 (see Figure 2): Suppose that the transition function specifies that the state moves to q_2 just after player 1's rejection of player 2's acceptance. If the machines are used, the split at period $t+1$ (odd period) is

$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right).$$

If, for instance, only player 1's transition function is changed (i.e., player 1 moves to q_2 after the first sub-period of period t), then the split is either

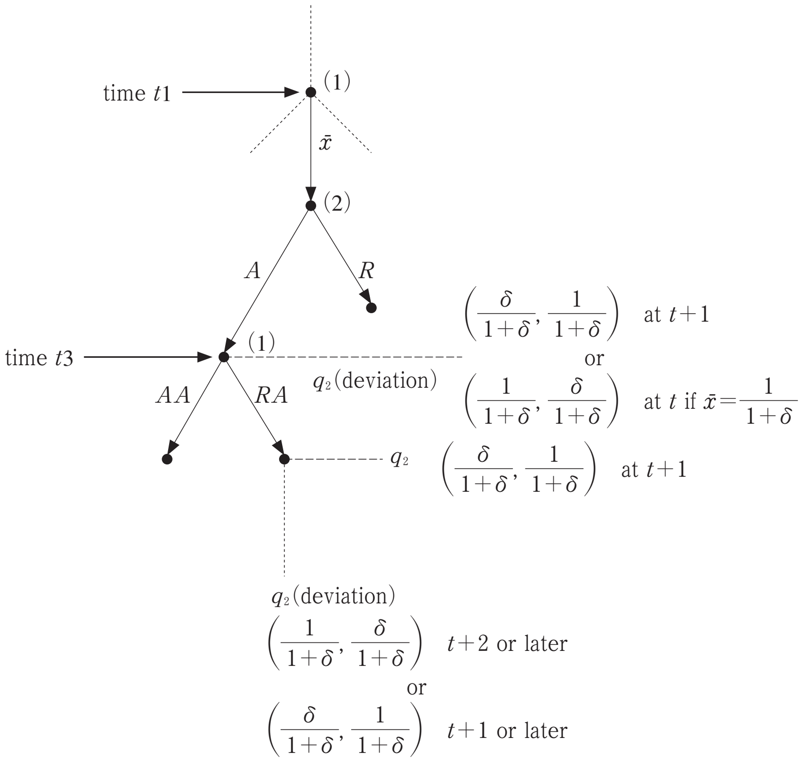
$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right) \text{ at period } t+1 \text{ or later, or}$$

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t+2 \text{ or later.}$$

By deviating, player 1 gets at most $\delta/(1+\delta)$ (evaluated at period $t+1$). Thus

(8) If the states are used more than once on the equilibrium path, we have to repeat the following argument.

Figure 2: Case 1



the deviations are not profitable.

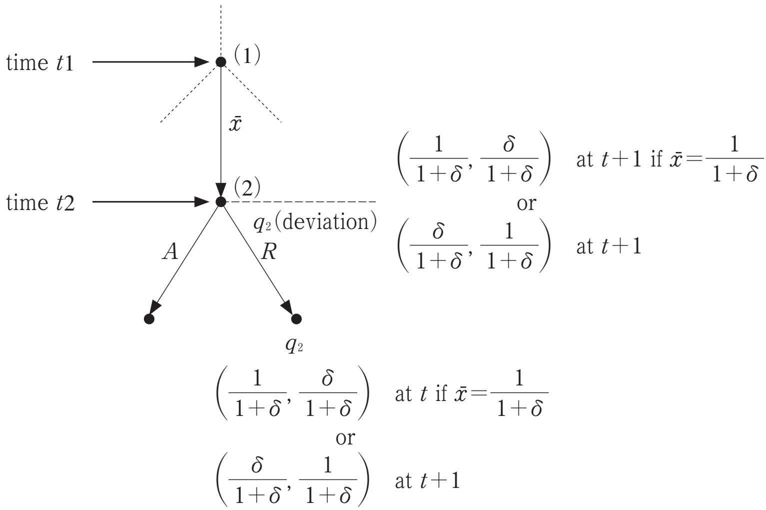
If, for instance, only player 1's transition function is changed (i.e., player 1 moves to q_2 before player 1's action at time $t3$), then the split is either

$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right) \text{ at period } t+1, \text{ or}$$

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t \text{ if } \bar{x}=\frac{1}{1+\delta}.$$

From the deviation, player 1 gets at least $\delta/(1+\delta)$ (evaluated at period $t+1$).

Figure 3: Case 2



Thus the deviation is profitable. So the state change does not occur just after player 1's rejection of player 2's acceptance.

Case 2 (see Figure 3): Suppose that the transition function specifies that the state moves after player 2's decision (A or R) at $t2$. If the machines are used, the split is either

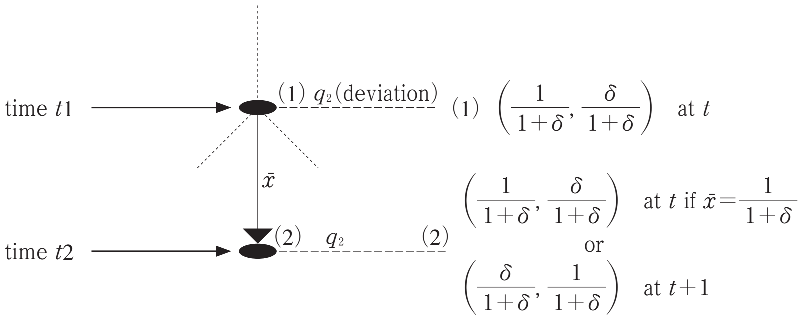
$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right) \text{ at period } t+1 \text{ or}$$

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t \text{ if } \bar{x}=\frac{1}{1+\delta}.$$

If, for instance, only player 1's transition function is changed (i.e., player 1 moves to q_2 before player 2's action at time $t2$), then the split is either

$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right) \text{ at period } t+1, \text{ or}$$

Figure 4: Case 3



$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t \text{ if } \bar{x}=\frac{1}{1+\delta}.$$

Thus the deviation can be profitable. Therefore the state change does not occur after player 2's move at time $t2$.

Case 3 (see Figure 4): Suppose that the transition function specifies that the state moves after player 1's offer. If the machines are used, the split is either

$$\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right) \text{ at period } t+1, \text{ or}$$

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t \text{ if } \bar{x}=\frac{1}{1+\delta}.$$

If, for instance, only player 1's transition function is changed (player 1 moves to q_2 at $t1$ but before player 1's move at time $t1$), then the split is

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \text{ at period } t.$$

By the deviation, player 1 gets $1/(1+\delta)$ (evaluated at period t). Thus the deviation is profitable. Thus the state change does not occur just after player 1's move at time $t1$.

By backward induction, player 1 must choose $1/(1+\delta)$ at time 01 in possible

Table 6: Possible ASPE Machines

		state q_1 (q_1', q_1'')	state q_2
M_1	demands	$x=\frac{1}{1+\delta}$	$\frac{1}{1+\delta}$
	accepts	—	$x\geq\frac{\delta}{1+\delta}$
	accepts the acceptance	—	$x\geq\frac{\delta^2}{1+\delta}$
M_2	proposes	—	$\frac{\delta}{1+\delta}$
	accepts	—	$x\leq\frac{1}{1+\delta}$
	accepts the acceptance	—	$(1-x)\geq\frac{\delta^2}{1+\delta}$
transitions		After player 1 offers, go to q_2	Absording

Note: "—" means any action.

ASPE machines in Table 6. Both players move to q_2 after player 1's offer. Notice that the transition function requires at least one input. Using ASPE, player 1, for example, can eliminate the state q_1 by replacing it by q_2. Thus there is no ASPE with essentially two states.

We can extend the above results to the case of more than two states by decomposing the game depending on the states. The analysis ends in finite time because we have assumed finite automata. Thus with the ASPE solution concept, the equilibrium machine induced from the bargaining game must use only one state. The equilibrium utilities for players are then the same as Rubinstein's. □

4 Conclusion

Although there are many subgame perfect equilibria in Muthoo's bargaining model, they are not equivalent in terms of the structures of strategies. A stationary strategy is often assumed in game-theoretic analysis when there are many equilibria or when it is very complicated to construct non-stationary strategies. In general, such an assumption is discouraged. This is because this assumption restricts players' freedom to choose strategies. In our model, guaranteeing freedom to choose strategies, the (induced) stationary equilibrium strategies arise endogenously from the machine game. Equilibrium strategies in the bargaining that are eliminated when complexity is taken account can be interpreted as unstable with respect to the specification of the players' preferences.

There are some issues with which we have not been concerned here. First, we also have multiple equilibrium in two-good bargaining. To support some equilibria we need a large number of states when the number of goods increase because a new state is needed for constructing subgame perfect equilibrium at the subgame where only one good remains. However, by considering trade-offs between achieving the highest payoff and minimizing the cost of implementation, we could also eliminate some of these equilibria. Second, we did not discuss the implementability of our machines. As proper machines, we need to specify what machines can know and recognize during the play in detail. Binmore et al. (1998) use proper machines to analyze bargaining models. There, however, states are allocated for proposal states, acceptance states, and acceptance of acceptance states. If we use such machines, we need more states than here even in the case of stationary machines.

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